

# AN ARMATURE-STATOR CONTACT RESISTANCE MODEL FOR EXPLOSIVELY DRIVEN HELICAL MAGNETIC FLUX COMPRESSION GENERATORS \*

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## Abstract

Although helical magnetic flux compression generators (HFCGs) have been in use for more than four decades, no one has been able to satisfactorily model their behavior. To bring computed currents into agreement with experimental values, tuning factors or anomalous flux loss factors are used. Such factors are not universal, and they must be adjusted for each generator design, or for different operational parameters (e.g., seed current or load inductance) for a given design. Many HFCG modeling codes have been reported over the years with various types of these empirical factors.

One of the recognized issues for HFCGs is magnetic flux loss near the moving contact point between expanding armature and helical stator coil winding. In our new model, we have analytically estimated the rate of magnetic field diffusion in the vicinity of the contact point. When converted to a flux loss rate, we find that it usually scales nonlinearly with the instantaneous current, and that the resulting effective resistance is proportional to the square root of the current. This result applies even at relatively small operating currents. Whereas the usual HFCG resistances drop as the generator length decreases, the contact resistance generally increases throughout operation. While small initially, we find that it usually dominates late in time and ultimately limits the gain of most generators.

In this paper, we present the derivation of the contact resistance model and show its effectiveness in estimating current gain for simple HFCG designs using a simple spreadsheet program. The model has also been implemented in the 1½-D FCG-model code, CAGEN, and an accompanying paper presents CAGEN results for a wide range of HFCGs, benchmarking the new model. The formulation for our model is universal; i.e., there are no adjustable factors, and it has generally enabled calculation of HFCG currents to within 20% of experimentally reported values.

## I. INTRODUCTION

### A. Helical Flux Compression Generators

Helical magnetic flux compression generators (HFCGs) have been in use for almost 50 years. They consist of two main components, a helical coil usually made from round-cross-section wire ("stator"), and an interior coaxial conducting cylinder ("armature") filled with high explosive (HE). The armature HE is typically detonated at one end, causing the armature to expand in a cone shape as the detonation wave proceeds down the axis. Current runs through the stator to an attached load and returns through the armature. Fig. 1 is a two-dimensional representation of a variable-pitch HFCG that shows magnetic streamlines for the initial state (upper) and later (lower) state, when the expanding armature makes contact with the stator. The generator initial inductance can be quite high, due to the flux lines linking almost all of the turns. As the generator functions, the moving contact point sweeps the flux ahead, removing the linkages and decreasing the effective inductance.

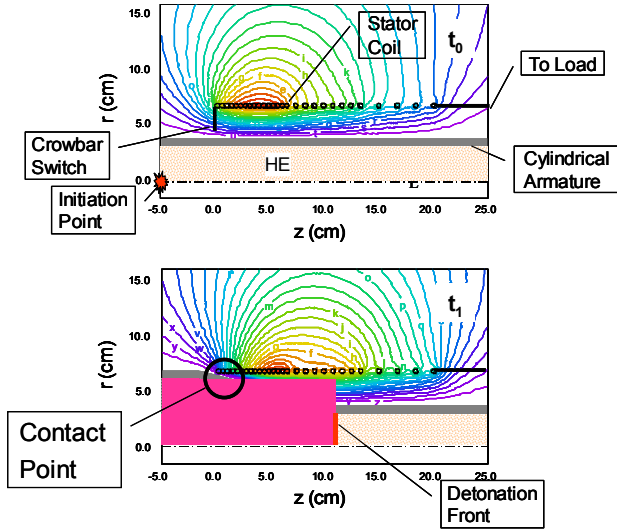
For modeling purposes, the FCG and its load are often approximated by a lumped-element circuit, such as shown in Fig. 2. Here, the FCG appears as a time-varying inductance and resistance. The circuit is initially charged with current,  $I_0$ , or "seed current," by a separate circuit, which is not shown. The time-varying inductance can be approximated by one of several techniques. The resistance should, in general, include diffusion effects (nonlinear, when the currents get large) and proximity effects, i.e., increase in resistance associated with non-uniform magnetic field distribution around each wire.

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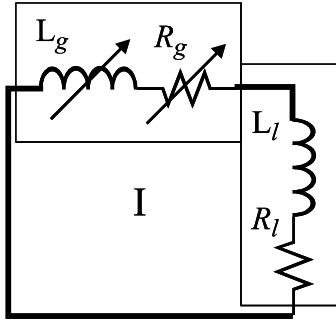
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**Figure 1.** Two-dimensional representation of a variable-pitch HFCG showing magnetic streamlines. Initial configuration (upper) at time  $t_0$  when detonation begins; configuration just as armature makes contact with stator (lower) at time  $t_1$ .



**Figure 2.** Schematic diagram for the equivalent circuit for an FCG and its load.

### B. The Generator Equation

Kirchoff's Voltage Law can be derived for the circuit of Fig. 2 by integrating Faraday's Law,

$$\frac{d(LI)}{dt} + RI = 0, \quad (1)$$

expanding to

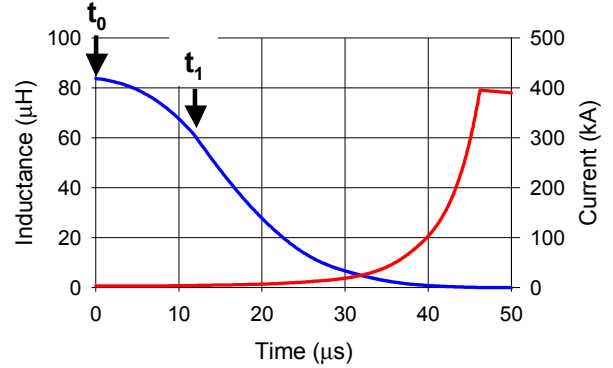
$$L\dot{I} + (\dot{L} + R)I = 0, \quad (2)$$

and integrating to yield the so-called Generator Equation:

$$L(t)I(t) = \psi(t) = \psi_0 \exp\left(\int_0^t \frac{R(t')}{L(t')} dt'\right). \quad (3)$$

Eq. (2) shows that current amplification ( $\dot{I} > 0$ ) can only occur if the magnitude of the time derivative of the inductance (negative quantity) is greater than the total series resistance. Assuming that the inductance and resistance are known (calculable), then Eq. (3) gives the

instantaneous flux, which, when divided by the total (generator plus load) inductance, gives the instantaneous current. Fig. 3 shows the time-dependent inductance and current amplification corresponding to the FCG in Fig. 1.



**Figure 3.** Time-dependent FCG inductance and associated current amplification. The times shown correspond to the times marked in Fig. 1.

The problem with the Generator Equation is that when modelers have used their best estimates of inductance and resistance, they have invariably over-predicted peak currents—typically by a factor of two or more. In order to bring the calculated peak currents into agreement with experimental data, they have done things such as artificially increase the load inductance or resistance, assumed that the armature does not actually contact the stator windings (leaving flux in the gap), imposed some internal voltage breakdown limit, or assumed that the resistance is just a fixed fraction of  $dL/dt$ . In all cases, the “correction” has had to be done empirically and changed or “tuned” to each generator design, or even to different operating parameters for a given design. Indeed, as Neuber and Dickens stated in a recent review of FCGs [1], “*But even if the inductance and resistance are accurately calculated, the treatment of the flux, diffused into the conductors and only partially recovered for further compression, poses a problem such that no a priori prediction of HFCG performance can currently be made. Merely an educated guess is possible which will greatly depend on the experience of the user. This means that the user will have to adjust at least one parameter in the code that is not based on physical principles but merely accounts in a heuristic way for the intrinsic flux loss.*”

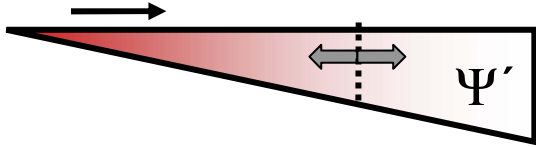
In this paper, we show that one does not need to invoke any “anomalous” or adjustable flux loss factors to achieve agreement with experiment if one can properly approximate the rate of flux diffusion into the conductors in the vicinity of the armature-stator (a-s) contact point. We present, in the next section, the derivation of our physics-based contact resistance model (KCRM). The final section describes a simple spreadsheet model for the Generator Equation, which includes the KCRM, and shows comparisons of calculation results to a limited set of experimental data.

## II. KCRM Derivation

### A. Basic Notion

Along the armature, moving conductive material attempts to push ahead the magnetic flux. However, because of the finite conductivity, flux also diffuses into the armature and the stator. The processes are inherently three-dimensional, and generally must be described in the magnetohydrodynamic (MHD) sense. However, no existing codes can handle the fine zoning required to resolve the diffusion regions as well as the large volume of the overall problem while including all the important physical processes in 3-D.

Our approach is rather to examine the diffusion process in the contact region, deduce an effective flux diffusion rate there, and add the loss rate to the other, more easily computed resistances for the rest of the circuit. We postulate that there is always a location in the generator, which we shall call the critical point, for which most of the flux behind diffuses into the conductors, and for which most of the flux ahead is advected ahead toward the load, as illustrated schematically in Fig. 4. If one can determine the flux-per-unit-length in the armature-stator gap at the critical point and multiply it by the velocity of the critical point,  $v_{cp}$ , then that quantity is the effective voltage across the generator at that point, and the resistance associated with it is just the voltage divided by the current. All of the subsequent derivation follows from various discussions (and references contained therein) in the classic book by Knoepfel [2]. Thus, we must determine the location of the critical point, its velocity, and the flux-per-unit-length there.



**Figure 4.** Looking into the gap between stator and armature, as it would appear if the stator were unwound. The magnetic field is emerging out of the page. The dotted line shows the location of the critical point, and the shading illustrates, qualitatively, the field magnitude.

The dimensionless quantity that describes the relative importance of flux advection compared to diffusion is the Magnetic Reynolds Number,  $R_m$ . It is given by the ratio of time to move flux over a given distance in vacuum to the time to diffuse the flux into a resistive medium through the same distance,

$$R_m = \frac{l^2/D_m}{l/v} = \frac{lv}{D_m}, \quad (4)$$

where the magnetic diffusivity,  $D_m = \eta/\mu_0 \approx 0.02$  m/s for copper or aluminum,  $\eta$  is the material resistivity, and  $\mu_0$  is the permeability of free space. For most of the generator the a-s gap distances are large and the armature

expansion velocity is on the order of  $10^3$  m/s, so that  $R_m \gg 1$ , and diffusion is relatively unimportant. However, when  $R_m \sim 1$ , flux diffuses about as rapidly as it is pushed ahead by the moving conductor.

### B. Important Relations

There will always be some time at a particular axial position within the generator (or, equivalently, some position in the generator at a time after armature-stator contact has first occurred) where the a-s gap distance  $g$  corresponds to  $R_m \sim 1$ . We therefore set the critical gap distance by the Magnetic Reynolds Number,

$$g \approx \frac{\eta R_m}{\mu_0 v}. \quad (5)$$

It turns out that critical gap distances are small, of order  $10^{-4}$  m. At these gap distances there are strong armature-stator proximity effects, reducing the importance of adjacent wires in the stator, and making the surface fields very strong and causing diffusion to be nonlinear.

The nonlinear resistivity can be approximated by

$$\eta = \eta_0(1 + \beta Q), \quad (6)$$

where  $\eta_0$  is the reference resistivity,  $Q$  is the specific energy (J/kg), and  $\beta$  is the “temperature” coefficient.

Assuming, for now, that the diffusion will be nonlinear, we use an expression that relates the specific energy in the material at the surface to the magnetic energy density just outside the conductors [3],

$$Q = \frac{B^2}{3\mu_0\rho}. \quad (7)$$

Note that this quantity is  $\frac{1}{2}$  of the usual magnetic energy density divided by the material density, which we assume is the ambient value. In the same reference [3], the nonlinear skin depth is given as

$$\delta_{nl} = \frac{\delta_0}{2} \frac{H_s}{H_{ch}}, \quad (8)$$

where  $H_s$  is the surface magnetic field, the characteristic field (i.e., the field value for which the surface resistivity has doubled from ambient) is given by [3]

$$H_{ch} = \sqrt{\frac{2}{\mu_0\beta}}, \quad (9)$$

and the skin layer approximation (approximately valid for quasi-exponentially rising fields) for the skin depth is given by

$$\delta_0 \approx \sqrt{\frac{\eta_0\tau}{\mu_0}}. \quad (10)$$

Here, the effective exponentiation time is approximated by

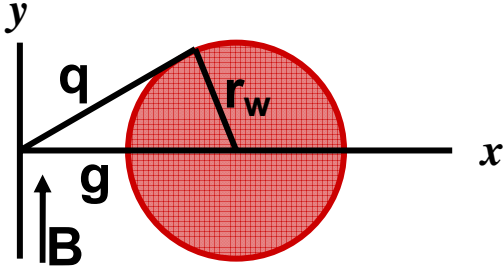
$$\tau \approx \frac{H_s}{\dot{H}_s} \approx \frac{2g}{v_\perp}, \quad (11)$$

which can be derived from Eq. (12), to follow. Here,  $v_\perp$  is the gap closure speed.

Next, to estimate the field in the gap, we approximate the geometry as Cartesian, because  $\Delta r/r$  is usually very small, and assume that the skin depth is also very small, because the effective magnetic field time derivative is very high as the two conductors approach each other. Finally, we assume that the proximity effect due to adjacent wires is negligible. We then look for the potential solution for the flux density in the gap using a conformal transformation for the geometry of Fig. 5. We find with these approximations that the vertical component of the flux density in the gap is given by

$$B_y(x) = \frac{\mu_0 I q}{\pi(q^2 - x^2)}, \quad (12)$$

where  $q^2 = (r_w + g)^2 - r_w^2 = 2r_w g + g^2$  is the square of the tangential distance from the armature ‘plane’ to the wire surface,  $r_w$  is the wire radius, and  $g$  is the gap distance.



**Figure 5.** Geometry for approximating the magnetic flux density in the a-s gap.

Equation (12) can be integrated analytically to obtain the flux per unit length in the gap. The result is

$$\Psi' = \int_0^g B_y dx = \frac{\mu_0 I}{2\pi} \ln \frac{(q+g)}{(q-g)}. \quad (13)$$

Since  $g \ll a$ , the logarithmic term can be approximated by its binomial expansion for simplification.

### C. Final Form and Discussion

When Eqs. (5)–(13) are combined, we obtain the following simplified expression for the magnetic flux per unit length of stator winding at the critical point:

$$\Psi' \approx \frac{\mu_0}{(2r_w)^{3/4}} \left( \frac{I}{\pi} \right)^{3/2} \left( \frac{\eta_0 \beta}{9v_\Delta \sin \alpha} \right)^{1/4}. \quad (14)$$

Here,  $v_\Delta$  is the HE detonation velocity, and  $\alpha$  is the armature cone half-angle.

The total rate of flux diffusion associated with this model is approximately the flux per unit length at the critical point ( $R_m = 1$ ) times the contact point velocity, since we assume that the critical point velocity, with respect to the contact point velocity, is small. The contact point velocity for fully developed flow is

$$v_{cp} = v_\Delta \sqrt{(2\pi_s n)^2 + 1} \approx 2\pi_s n v_\Delta. \quad (15)$$

Here,  $r_s$  is the coil inside major radius and  $n$  is the inverse coil pitch (turns per unit length).

We finally obtain, with the aforementioned assumptions, the (simplified) analytic form for the nonlinear contact resistance:

$$R_{cp} \approx \mu_0 n r_s \left( \frac{I}{\pi} \right)^{1/2} \left( \frac{v_\Delta}{r_w} \right)^{3/4} \left( \frac{2\eta_0 \beta}{3 \sin \alpha} \right)^{1/4}. \quad (16)$$

Equation (16) shows that the contact point resistance is intrinsically nonlinear and scales as the square root of the current. It depends only weakly on the material properties and the armature cone expansion angle. The expression can be generalized to accommodate different armature and stator materials, as well as include the limiting low current case.

## III. Calculations and Comparison to Experiment

### A. Spreadsheet Model

In order to test the KCRM, we set up a relatively simple spreadsheet computational model. The time-dependent generator inductance is calculated off-line using a generalization of a formula in Smythe [4], and the resulting table is imported into the spreadsheet. We assume that all resistance in the circuit, except the contact resistance, is linear and given by the product of resistivity and conductor length, and divided by cross-sectional area. In the following, we show the effects of different forms of the resistance. We use the skin layer approximation for the penetration of the current into the conductors, as in Eq. (10). For the stator, we also include a proximity factor [5]. The contact resistance (Eq. (16)) is added to obtain the total resistance, and Eq. (1) is integrated explicitly.

### B. Results

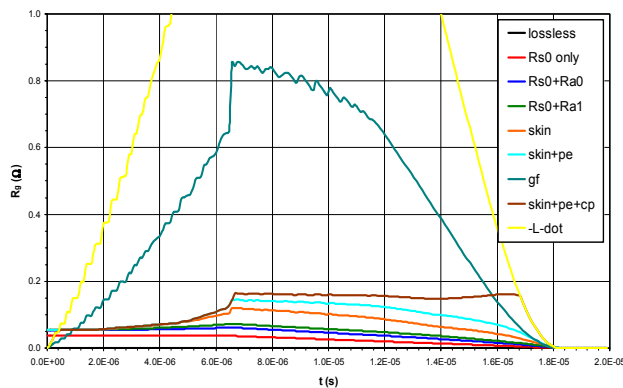
To illustrate the effect of resistance on calculated generator performance, we use a very simple helical generator, the TTU-1 [6]. The generator consists of a 32-turn stator winding of insulated 12 AWG copper wire

with a pitch of 3 mm, for an overall length of 96 mm. Its inside radius is 38 mm. The armature is made of either aluminum or copper with outer radius of 19 mm and wall thickness of either 2 mm or 3 mm. The interior is packed with C-4 explosive with detonation speed approximately 8200 m/s. For the calculations, we assumed an expansion cone half-angle of 13.5°. The load inductance was 46 nH.

Figure 6 shows the results of calculations for the time-dependent linear (constant resistivity) resistance of the generator. Of course, if the generator is "lossless" there is no resistance. Next, we show the case of dc stator resistance only ("Rs0"). This resistance is proportional to the instantaneous stator length (i.e., ahead of the contact point). In fact, all of the linear resistance approximations have this direct length dependence. The next curve ("Rs0+Ra0") adds in the dc armature resistance, which takes into consideration variable armature radius and thickness in the expansion region. The next curve ("Rs0+Ra1") includes an estimate of the effects of initial armature shock heating and plastic work heating during expansion [7]. These effects are seen to be relatively small. The next curve ("skin") includes the dynamic skin effect. The current is assumed to be constant over a depth given by Eq. (10), where the time constant  $\tau$  is approximated by the instantaneous value of  $I/i$ . The next curve ("skin+pe") adds in an estimate of the wire-to-wire proximity effect for the stator [5]. Both the skin and proximity effects are seen to be rather significant.

The curve designated "gf" represents a non-physical resistance based on a constant fraction of the generator inductance time derivative. The constant was adjusted to give the same peak current as was obtained experimentally. It is seen to be much greater than the estimates of the linear resistances, underscoring their inability to account for the actual generator losses.

For reference, the uppermost curve ("-L-dot") shows the instantaneous rate of inductance change for the generator. Note that it is quite large, leading to large ideal current amplification, and that it exceeds the previous resistances at all times.

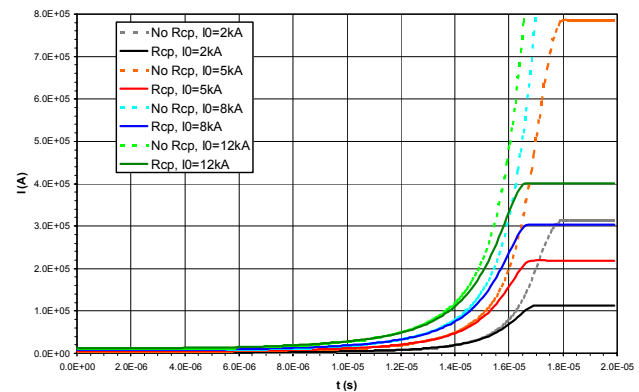


**Figure 6.** Summary of resistance calculations for the TTU-1 generator.

The final curve, "skin+pe+cp," includes the resistance associated with the nonlinear diffusion in the vicinity of

the contact point as described in the previous section. Since its value implicitly depends on current, we chose an initial current of 5 kA for this calculation. One can see that its incremental value increases in time after it starts when a-s contact first occurs. It is initially a relatively small fraction of the total resistance, but since the other components are monotonically decreasing with time, its relative importance increases. In fact, unlike the resistance without this component, the total resistance with the contact resistance eventually equals  $dL/dt$  before the end of generator operation. Remember that from Eq. (2), current amplification ceases when this occurs.

In Fig. 7, we show the effect of the previously presented resistances on estimation of current. The graph shows the calculated currents, with and without inclusion of the contact resistance, for a variety of seed currents. One can see from the figure that inclusion of the relatively small contact resistance dramatically changes the calculated peak current. The lowest seed current case, 2 kA, where nonlinear effects are not predicted, exhibits a reduction of the peak current by approximately a factor of three, and the reduction factor increases for increasing seed current.

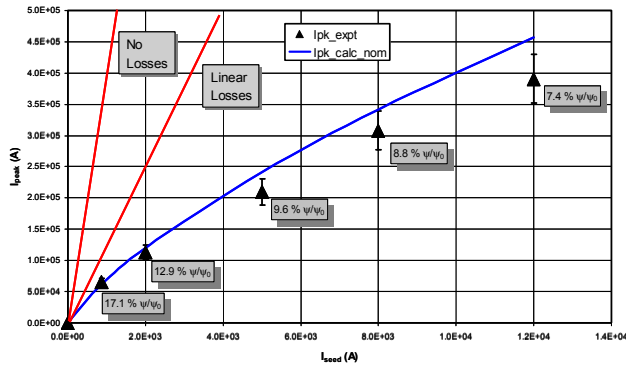


**Figure 7.** Effect of contact resistance on calculated TTU-1 generator currents.

Figure 8 shows a summary of the TTU-1 spreadsheet calculation results compared to the published experimental data. Again, there was no empirical adjustment of any circuit parameters in these calculations. For reference we show the prediction for peak current scaling for both ideal and best linear resistance cases. The agreement is quite good, and would perhaps been even better had we been able to include nonlinear diffusion throughout the generator and stator deformation due to magnetic pressure [8].

We conclude this section with a comparison of the new model calculations with detailed data for a medium-sized FCG operated at modest parameters [9]. While still a simple design, this generator represents the limit of the capability of the spreadsheet model described above, and in fact the spreadsheet algorithm for the total resistance,



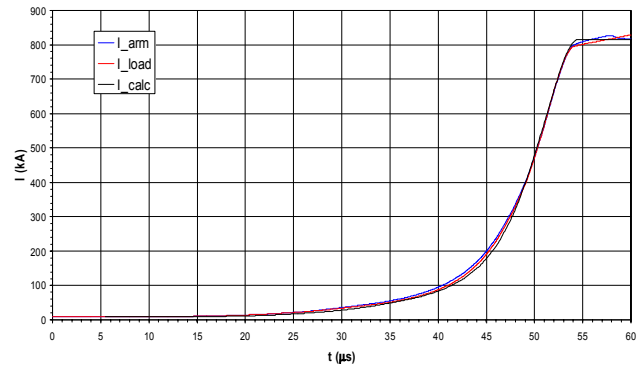


**Figure 8.** Summary of calculations of peak current and comparison to experimental data for the TTU-1 FCG. The triangles are experimental data points, the thin continuous line is the present calculation including contact resistance. The two straight lines represent the ideal (lossless) case and the best linear resistance estimate without the contact contribution.

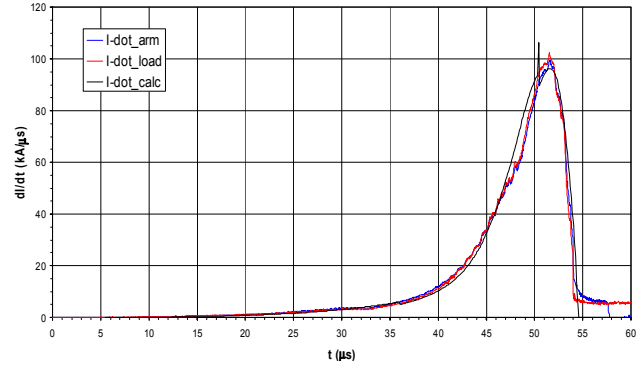
including contact resistance, was imported into MathCad™ for this calculation. However, the contact resistance model was not changed in any way. Figure 9 shows a comparison of the current as calculated with the model to current from two different Rogowski coil measurements. The similarity of the two independent measurement curves indicates the integrity of the FCG operation and measurement technique. The model calculations essentially overlay both traces. An even more stringent comparison, of the current time derivative, is shown in Fig. 10. Again, the degree of agreement between experiment and model is excellent.

#### IV. SUMMARY

We have described a new model for the resistance associated with the armature-stator contact point in helical flux compression generators. The model is based on analytic approximations to the nonlinear diffusion of fields in the gap at a critical point, behind which all of the flux is assumed to diffuse into the conductors rather than advect forward toward the load. The model shows that because of the inherent nonlinearity of the diffusion process, the effective contact resistance scales as the square root of the instantaneous current and depends only weakly on generator material properties. Incorporation of the model into a simple spreadsheet program shows that while the contact resistance is small, its value increases while the usual armature and stator resistance values decrease. Eventually, it not only dominates the total resistance, but also becomes equal in magnitude to the generator inductance derivative, thus limiting the peak gain. The model has shown excellent agreement with peak versus seed current scaling for a small, simple FCG, as well as with detailed current and current derivative for a medium-size generator.



**Figure 9.** Comparison of FCG current measured with armature and load Rogowski coils to model calculation.



**Figure 10.** Comparison of current time derivative for Fig. 9. The calculation is the smooth curve.

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